

positional rigid-body parameters are explicitly evaluated in order to permit a simultaneous refinement of these parameters. It is also possible to apply only geometrical or only thermal constraints and treat a part of the structure in an unconstrained manner in which cases the derivatives are appropriately modified.

The general organization of the program was based in part on an *ORFLS* version modified by Professor F. L. Hirshfeld for a purpose similar to ours, but with some constraints different.

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The Crystal Structure of Sodium Diborate $\text{Na}_2\text{O} \cdot 2\text{B}_2\text{O}_3$

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Sodium diborate, $\text{Na}_2\text{O} \cdot 2\text{B}_2\text{O}_3$, is triclinic, space group $P\bar{1}$, with unit-cell dimensions at 22°C: $a = 6.5445 \pm 0.0004$, $b = 8.6205 \pm 0.0004$, $c = 10.4855 \pm 0.0006$ Å, $\alpha = 93.279 \pm 0.006$, $\beta = 94.870 \pm 0.006$, $\gamma = 90.843 \pm 0.006^\circ$. The calculated density is 2.272 g cm^{-3} with four formula units in the cell. The structure was determined from three-dimensional X-ray data by direct methods. A full-matrix least-squares refinement resulted in an R index of 0.035 (0.040 for the weighted R index). The anion borate polymer in this structure forms layers composed of di-pentaborate groups and triborate groups with one non-bridging oxygen. This is a novel feature in diborates. Boron–oxygen bond lengths are normal, but with significant differences depending on the location within the groups. The sodium atoms are coordinated by six or seven oxygen atoms at distances ranging from 2.260 to 2.872 Å.

Introduction

Several of the anhydrous diborates have polymer structures built up from diborate groups. This is the case for lithium diborate, $\text{Li}_2\text{O} \cdot 2\text{B}_2\text{O}_3$ (Krogh-Moe, 1968), and cadmium diborate, $\text{CdO} \cdot 2\text{B}_2\text{O}_3$ (Ihara & Krogh-Moe, 1966). The structure of this cadmium compound is the first determined structure in an isomorphous series including the Mg, Mn, Fe, Co, Ni, Zn, Hg compounds (Ecker, 1966). Diborates containing groups other than the diborate group are known, however. Potassium diborate (Krogh-Moe, 1972*a*) has, in addition to diborate groups, di-triborate groups. Barium diborate (Block & Perloff, 1965) has a single framework based on di-triborate and di-pentaborate groups. The structure reported for sodium diborate in the

present paper provides yet another example of a diborate with a novel combination of groups.

Sodium diborate exists in more than one modification (Jenkel, 1936; Milman & Bouaziz, 1968). The phase studied here crystallizes from the melt (or glass) below the melting point 742°C down to about 650°C. It appears to be the stable phase, at least in the temperature range mentioned. Vila & Font-Altaba (1955) have reported unit-cell dimensions for what appears to be the same phase, though their values differ appreciably from those obtained by us.

Experimental

Anhydrous sodium diborate was prepared by dehydrating and melting borax, $\text{Na}_2\text{O} \cdot 2\text{B}_2\text{O}_3 \cdot 10\text{H}_2\text{O}$, (p.a.

Table 1. Final observed and calculated structure factors

The columns are I, 10F_o and 10F_c. The reflexions marked by asterisks are affected by extinction and were omitted from the final cycles of least-square refinement.

Table with multiple columns containing numerical data for structure factors. The columns are labeled with Miller indices (hkl) such as 0 0 0, 1 0 0, 2 0 0, etc. The data includes observed values (I), observed structure factors (10F_o), and calculated structure factors (10F_c). Asterisks are used to mark certain data points.

quality) in a platinum crucible at 800°C . The viscous liquid was then crystallized at a temperature of about 700°C . A single crystal with a prismatic shape and dimensions $0.008 \times 0.009 \times 0.014$ cm was used for the X-ray work. Intensity data were collected with an on-line Picker single-crystal automatic diffractometer and $\text{Cu K}\alpha$ radiation. The X-ray reflexions were measured

at 2106 different reciprocal lattice points. 2079 reflexions were observed larger than the background.

Unit-cell dimensions and standard errors are: $a = 6.5445 \pm 0.0004$, $b = 8.6205 \pm 0.0004$, $c = 10.4855 \pm 0.0006$ Å, $\alpha = 93.279 \pm 0.006$, $\beta = 94.870 \pm 0.006$, $\gamma = 90.843 \pm 0.006^\circ$, and were obtained by the method of least squares from angle data recorded at 22°C for 12

Table 2. Final atomic parameters

The anisotropic temperature factors are of the form $\exp[-2\pi^2(U_{11}(ha^*)^2 + U_{22}(kb^*)^2 + U_{33}(lc^*)^2 + 2U_{12}hka^*b^* + 2U_{13}hla^*c^* + 2U_{23}k/b^*c^*)]$ corresponding to the Cruickshank (1956) U notation. Positional parameters are expressed as fractions of the cell edges. All values are multiplied by 10^4 .

	x	y	z	U_{11}	U_{22}	U_{33}	U_{12}	U_{13}	U_{23}
Na(1)	1099	1331	6123	158	467	167	83	71	96
Na(2)	1751	3394	0176	143	257	234	28	-1	-27
Na(3)	4773	4643	2272	283	285	235	167	-41	-85
Na(4)	4393	9221	8562	206	184	104	-73	20	-3
O(1)	1481	0992	1548	128	65	103	-10	59	3
O(2)	1952	1369	3899	63	173	99	-16	7	35
O(3)	0992	3453	2499	88	65	156	-4	50	8
O(4)	1393	8652	7119	50	86	88	-19	10	-8
O(5)	1987	7772	4918	81	167	70	-1	14	11
O(6)	2162	5973	6594	104	85	164	13	76	28
O(7)	4719	7946	6649	53	178	83	-9	10	21
O(8)	3089	6936	2876	80	196	90	15	10	-30
O(9)	0347	8417	1884	111	68	166	12	70	24
O(10)	1859	6245	0754	120	79	76	-20	28	-11
O(11)	0480	4043	7574	93	82	178	10	58	37
O(12)	4509	2033	5527	87	323	88	-37	14	-9
O(13)	3039	8873	0559	132	90	132	-8	75	-5
O(14)	4746	6812	9551	150	137	154	35	82	21
B(1)	0709	1770	2713	71	92	94	0	23	-2
B(2)	2494	7605	6317	86	109	101	-11	14	26
B(3)	1178	6847	2005	94	89	83	-1	17	8
B(4)	3290	7225	0254	97	111	84	11	-16	3
B(5)	1555	9444	1335	107	120	81	2	0	11
B(6)	0512	5533	7202	100	104	84	-11	-5	-3
B(7)	3917	1807	4220	108	109	101	8	22	17
B(8)	3443	7561	4091	110	105	118	8	-1	32

Standard deviations									
Na	1.7	1.5	1	6	7	6	5	4	5
O	2.6	2	1.6	8	9	9	7	7	7
B	4.2	3.4	2.6	13	14	13	11	11	11

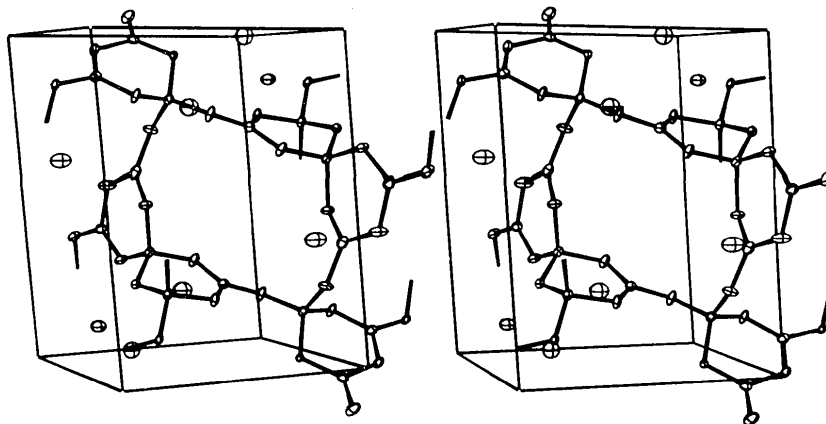


Fig. 1. The structure of sodium diborate, shown as a stereo-pair. The unit cell is indicated. The a axis is approximately perpendicular to the plane of the paper the b axis is horizontal and the c axis vertical in the plane of the paper. The boron-oxygen bonds are shown as solid lines. The isolated vibration ellipsoids represent the sodium atoms.

high-angle reflexions (based on the wavelength 1.5405 Å for Cu $K\alpha_1$). With four formula units of $\text{Na}_2\text{O} \cdot 2\text{B}_2\text{O}_3$ in the cell, the calculated density is 2.272 g cm^{-3} . Jenkel (1936) measured a density of 2.279 g cm^{-3} for this phase.

Structure determination

The observed intensities were converted to structure factors in the usual manner. A correction for absorption was also applied, the linear absorption coefficient being 31.3 cm^{-1} ($\lambda = 1.54178$).

A statistical test of the distribution of normalized structure factors gave a strong indication for the presence of a centre of symmetry. The structure was determined by direct methods, with the program system developed by Germain, Main & Woolfson (1971). The set of signs with the highest 'figure of merit' proved to give a sensible structure. The structure was refined by the method of least squares, with the X-RAY 71 program system edited by Stewart, Kruger, Kundell & Baldwin (1971). All the 2079 reflexions observed to be larger than the background were used in the refinement. The atomic scattering factors for O, B and Na^+ were taken from *International Tables for X-ray Crystallography* (1962). The refinement was carried out with a weighting scheme based on the statistical counting errors compounded with errors assumed to be 1% of the observed intensity. The structure refined to an R index of 0.045 (or 0.056 for the weighted R index). The 7 reflexions with the strongest intensities were systematically calculated larger than observed. These reflexions were assumed to be affected by extinction and removed from the data set. The data now refined to an R index of 0.035 (0.040 for the weighted R index). During the last cycle of refinement the largest shift to error ratio was 0.22. The observed and calculated struc-

ture factors are given in Table 1. (The reflexions affected by extinction are marked by an asterisk.) The final atomic coordinates and the thermal parameters are given in Table 2.

Discussion of the structure

The anion network in sodium diborate may be described as infinitely extending layers approximately perpendicular to the c axis. A layer structure in anhydrous borates has previously only been found for β -sodium triborate (Krogh-Moe, 1972*b*). A section of the borate layer in sodium diborate can be seen in Fig. 1 as a stereo-pair. The layer is composed of two types of borate groups, di-pentaborate groups [notation: see Krogh-Moe (1972*a*)] and triborate groups. These groups can be seen in Fig. 2. The di-pentaborate group was found by Block & Perloff (1965) in barium diborate. The triborate group found here is new for the diborate composition. It has one non-bridging oxygen, O(14). Non-bridging oxygens have not previously been found either in crystalline anhydrous borates of the diborate composition or in more acid compositions. Because of the presence of non-bridging oxygens, the fraction of boron atoms in fourfold coordination, $\frac{3}{8}$, is lower than the value, $\frac{1}{2}$, expected for anhydrous diborates (Krogh-Moe, 1960).

Boron-oxygen bond lengths and bond angles are reported in Table 3. Systematic deviations of bond lengths from average values, depending on the local surroundings of the bond, can be seen. In particular

Table 3. *Interatomic distances and bond angles*

Boron-oxygen bond lengths (standard deviation 0.003 Å), sodium-oxygen bond lengths (standard deviation 0.002 Å) below 3 Å and oxygen-boron-oxygen and boron-oxygen-boron bond angles (standard deviation 0.2°).

B(1)-O(1)	1.489 Å	Na(1)-O(9)	2.366 Å
B(1)-O(2)	1.488	Na(1)-O(5)	2.374
B(1)-O(3)	1.492	Na(1)-O(2)	2.445
B(1)-O(4)	1.446	Na(1)-O(12)	2.445
B(2)-O(4)	1.440	Na(1)-O(4)	2.592
B(2)-O(5)	1.492	Na(1)-O(11)	2.770
B(2)-O(6)	1.470	Na(2)-O(14)	2.297
B(2)-O(7)	1.489	Na(2)-O(10)	2.496
B(3)-O(8)	1.483	Na(2)-O(10')	2.509
B(3)-O(9)	1.474	Na(2)-O(3)	2.525
B(3)-O(10)	1.489	Na(2)-O(1)	2.600
B(3)-O(11)	1.434	Na(2)-O(9)	2.835
B(4)-O(10)	1.406	Na(2)-O(11)	2.872
B(4)-O(13)	1.453	Na(3)-O(14)	2.273
B(4)-O(14)	1.295	Na(3)-O(6)	2.328
B(5)-O(1)	1.343	Na(3)-O(8)	2.358
B(5)-O(9)	1.363	Na(3)-O(7)	2.572
B(5)-O(13)	1.396	Na(3)-O(3)	2.699
B(6)-O(3)	1.371	Na(3)-O(10)	2.809
B(6)-O(6)	1.360	Na(4)-O(7)	2.260
B(6)-O(11)	1.364	Na(4)-O(13)	2.374
B(7)-O(2)	1.344	Na(4)-O(14)	2.379
B(7)-O(7)	1.351	Na(4)-O(4)	2.402
B(7)-O(12)	1.395	Na(4)-O(13')	2.420
B(8)-O(5)	1.349	Na(4)-O(1)	2.722
B(8)-O(8)	1.355		
B(8)-O(12)	1.399,		

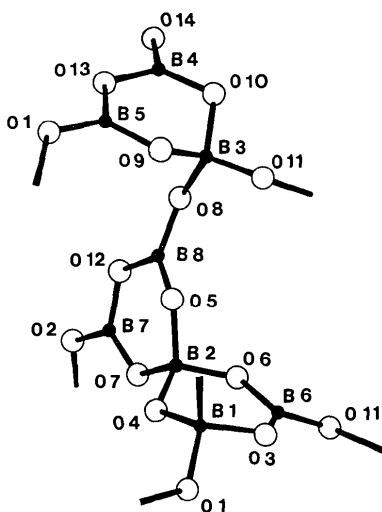


Fig. 2. The asymmetric unit of the boron-oxygen network, showing the triborate group and the di-pentaborate group. The atoms are numbered in accordance with Table 2.

Table 3 (cont.)

O(1)—B(1)—O(2)	111.5°	O(3)—B(6)—O(6)	122.3°
O(1)—B(1)—O(3)	102.8	O(3)—B(6)—O(11)	121.0
O(1)—B(1)—O(4)	113.2	O(6)—B(6)—O(11)	116.6
O(2)—B(1)—O(3)	110.0	O(2)—B(7)—O(7)	123.5
O(2)—B(1)—O(4)	106.6	O(2)—B(7)—O(12)	116.4
O(3)—B(1)—O(4)	112.9	O(7)—B(7)—O(12)	120.1
O(4)—B(2)—O(5)	113.4	O(5)—B(8)—O(8)	124.5
O(4)—B(2)—O(6)	112.0	O(5)—B(8)—O(12)	120.3
O(4)—B(2)—O(7)	106.9	O(8)—B(8)—O(12)	115.2
O(5)—B(2)—O(6)	108.7		
O(5)—B(2)—O(7)	109.5	B(1)—O(1)—B(5)	123.9
O(6)—B(2)—O(7)	106.0	B(1)—O(2)—B(7)	125.3
O(8)—B(3)—O(9)	109.5	B(1)—O(3)—B(6)	119.2
O(8)—B(3)—O(10)	104.0	B(1)—O(4)—B(2)	117.0
O(8)—B(3)—O(11)	115.7	B(2)—O(5)—B(8)	120.2
O(9)—B(3)—O(10)	110.0	B(2)—O(6)—B(6)	120.9
O(9)—B(3)—O(11)	104.2	B(2)—O(7)—B(7)	124.3
O(10)—B(3)—O(11)	113.5	B(3)—O(8)—B(8)	130.2
O(10)—B(4)—O(13)	114.8	B(3)—O(9)—B(5)	115.5
O(10)—B(4)—O(14)	127.1	B(3)—O(10)—B(4)	113.7
O(13)—B(4)—O(14)	118.1	B(3)—O(11)—B(6)	129.6
O(1)—B(5)—O(9)	123.7	B(7)—O(12)—B(8)	118.5
O(1)—B(5)—O(13)	117.3	B(4)—O(13)—B(5)	122.9
O(9)—B(5)—O(13)	119.0		

attention is called to the boron–oxygen bond between B(4) and O(14). This bond, involving the non-bridging oxygen O(14) is quite short, 1.295 Å. However, a comparable short bond length of 1.280 Å to the non-bridging oxygen in sodium metaborate has been found by Marezio, Plettinger & Zachariassen (1963). The other two boron–oxygen bonds involving B(4) (especially the B(4)—O(13) bond of 1.453 Å) are comparatively long. This effect is apparently induced by the short B(4)—O(14) bond.

With the exception of Na(2), which is coordinated by seven oxygen atoms in the range 2.297 to 2.872 Å, the sodium ions are coordinated by six oxygen atoms in the range 2.260 to 2.809 Å. If sodium–oxygen distances above 3 Å are not counted in the coordination

sphere, the coordination number six in the present case is the same as that observed in $\beta\text{-Na}_2\text{O} \cdot 3\text{B}_2\text{O}_3$ (Krogh-Moe, 1972b). Bond distances in these two compounds are also similar. The rather broad distribution in the sodium–oxygen bond distances is presumably typical of the borate network structures.

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